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Application of optimal adaptive generalized predictive control to a packed distillation column

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Abstract

In this work, optimal operating conditions for a packed distillation column and optimal adaptive generalized predictive control (OA-GPC) were investigated. Thus, the dynamic and steady-state properties of the packed distillation column distilling methanol–water mixture were observed experimentally and theoretically. Mathematical models for the packed distillation column were solved with orthogonal collocation on finite elements. Optimal operating conditions of the system were found by using Box–Wilson optimization method and "Experimental Design" technique.

Two types of control algorithm were utilized for controlling the packed distillation column, viz. conventional proportional integral derivative (PID) and generalized predictive control (GPC) at optimal operating conditions. Overhead temperature control was examined experimentally and theoretically. Pseudo random binary sequence (PRBS) signal and recursive identification algorithm were used to estimate the relevant parameters of the polynomial ARIMAX model. Generally theoretical and experimental control results were in accord with each other and it was observed that OA-GPC produced better performance than PID for the packed distillation column. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Packed distillation; Optimization; Generalized predictive control

1. Introduction

One of the most widely used unit operations in the process industries is distillation. It is a complex, multivariable, nonlinear dynamic system and the extent of the nonlinearities involved is ascertained by the range of operation of the distillation process. The complexity of distillation columns can range from a binary separation with constant molar overflow to a multicomponent nonideal separations with reaction and multiple feed and side steams. For a column to operate efficiently not only must it be designed well, but the control strategies executed must be effective as well. It is necessary to maintain the required controlled variables in the face of many disturbances that occur in industrial situations. The operation of distillation may be carried out either as a batch process or as a continuous one, and may be

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implemented in packed columns or in stage-wise contact towers. The present work is concerned with the continuous separation of a methanol–water mixture in a packed distillation column in a pilot plant. Packed columns can display high efficiencies and high capacities because of low resistance to liquid entraintment, low pressure drop and low liquid hold up (important for heat sensitive material that might decay at the base of the column) moreover the capital cost of packed columns is generally less than that of plate columns performing the same separation whereas the operational costs are about the same [1]. The purpose of the control is to keep the top product composition constant by using inferential control. Because of the difficulties faced in measuring the composition, the temperature of the top product is measured and controlled by means of GPC and the manipulated variable *Q*.

The conventional method of controlling processes is to apply a multiplicity of supposedly independent feedback control loops. Process control systems integrate adjustable controller settings that promote process operation over a wide range of condition. The simple three term proportional integral derivative (PID) and two term (PI) controllers remain the most generally applied industrial process controllers today. This is mainly due to the ease of operation, the robustness and the lack of specific process knowledge

Abbreivations: ARIMAX, autoreggressive integral moving average with external input; GMV, generalized minimum variance; GPC, generalized predictive control; IAE, integral absolute of error; ISE, integral square of error; OPT, optimum values; PID, proportional-integral-derivative; PRBS, pseudo random binary sequence

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Nomenclature

which is required for the initial controller design. The controller which provides the best performance may be selected by examining alternative P, PI, and PID combinations. PID control action was employed throughout-being considered the most likely type of control action for this application. The controller parameters were estimated using three different closed loop response tuning criteria for discrete controller viz. those developed by Yuwana and Seborg [2] denoted by YS, Jutan and Rodriques [3] denoted by YS-JR and Wardle and Heatchoch [4] denoted by YS-WH. An additional criterion (the YS-increase gain method) has been proposed in the present work.

Generally, three forms of self-tuning control have been encountered and applied. These are self-tuning PID (STPID) control, generalized minimum variance (GMV) control and generalized predictive control (GPC). The application

of GMV and STPID control systems have been examined by some researchers recently [5,6]. Optimal adaptive generalized predictive control (OA-GPC) means keeping distillation at optimal condition and GPC also provides a stable closed-loop response which minimize some type of quadratic cost function. OA-GPC is perceived to be superior to classical control systems. This receding horizon method predicts the plant's output over several sampling intervals using suppositions about future control actions [7]. GPC is claimed to overcome the following difficulties, nonminimum phase plant, open loop unstable plant, plant with variable or unknown dead-time and plants of unknown order. It is possible using the GPC to obtain stable control of processes with variable parameters, with variable dead-time, and with a model order which varies instantaneously provided that the input–output data are sufficient for reasonable plant identification.

The related work is avaliable in the previously published papers on the same packed distillation column. A lot of analyses on the process dynamics are achieved. The validity of the packed distillation column model was investigated by comparing experimental data very well. The steady-state and dynamic behavior of a binary packed distillation column has been simulated using two film back-mixing model. The model solution has been obtained employing orthogonal collocation on finite elements. As a result, the back-mixing model and simulation program represent the dynamic behavior of the packed distillation column separating methanol–water system with the thermosiphon reboiler. From the point above, the design of the GPC system such as tuning parameter, defining ARIMAX model parameters was evaluated using back-mixing model and related simulation program [8]. Pseudo random binary sequence (PRBS) signal and Bierman recursive identification algorithms were used to calculate the model parameters. It was shown that the dynamic simulation program can be used in the design of the GPC system to control the overhead temperature.

The purpose of this work is to investigate the optimal operating parameters of a packed distillation column and to study their effects on the overhead composition related with the temperature in the sense of equilibrium condition of the packed column by applying Box–Wilson optimization method which is based on experimental design. Reflux ratio and feed flow rate were chosen as very effective variables of overhead product composition. In experimental design, the objective function is to make the overhead product composition maximum by keeping the feed temperature and composition and reboiler heat duty and the heater oil flow rate at the desired operating values and determining the optimum values of reflux ratio and feed flow rate, when their values are between the highest and the lowest level. In that case only one set of parameters was determined for maximum overhead composition, even if there are an infinitive number of steady-state with the same distillate composition. Further knowledge can be found in related references [9].

In this work, only one set point optimization was used. When the set point changed, another optimal condition was obtained by using "Experimental Design" technique and a new optimal set point was achieved. It was intended to keep the top temperature at the desired value under the optimal condition by using OA-GPC and well tuned PID control systems. Comparison of OA-GPC system with PID control was investigated experimentally and theoretically. Several disturbances were given to the packed column and the performance of the GPC system was observed and compared with well tuned PID control. Conventional GPC control system gives better performance to control the overhead temperature than well tuned PID control system. Addition of adaptive optimization to the GPC algorithm makes control much more effective when the operating condition changes extremely.

2. Material and method

2.1. Experimental work

Physical properties of the packed distillation column used in the experimental studies were determined. Therefore, to obtain the constant amount of continuous bottom product, a "U" pipe is added to the bottom of the packed column. The reboiler was made from 131 glass container. For feeding relevant liquid into the column and oil container, two 1BG pumps which are suitable for chemicals were utilized. To heat the oil container and feed mixture, 4.5 kW triyac module and 2.5 kW electrical heaters are used, respectively. Reflux ratio was adjusted by using magnetic valve with a timer. The valve distributed the liquid obtained from the condenser as reflux and overhead product. The system temperature was measured with six thermocouples which were attached to the oil container, reboiler, connection point of the reboiler and the column, the middle of the column, top of column and feed. Each thermocouple was connected to a six selection digital channel. Temperature sensed by the selection channel was transferred to the computer with an A/D converter. Temperature data measured for each minute were recorded. Physical details of the packed distillation column are given in Table 1. All the experimental equipment is shown in Fig. 1.

Table 1 Physical details of the packed distillation column

| Packing height (mm) | 1400 |
|---------------------------------------|---------|
| Inside diameter of packed column (mm) | 80 |
| Packing type | Raschng |
| Packing diameter (mm) | 20/15 |
| Feed tank volume (1) | 60 |
| Reboiler volume (1) | 13 |
| Heater oil volume (1) | 25 |
| Total pressure (mmHg) | 690 |
| | |

Fig. 1. Experimental equipment: 1, big vessel; 2, packed column; 3, condenser; 4, temperature converter; 5, A/D converter; 6, refluxer; 7, magnetic valve; 8, computer; 9, D/A converter; 10, transducer; 11, control valve; 12, heat exchanger; 13, rotameter; 14, feed vessel; 15, pump; 16, triyac module; 17, oil tank; 18, cooling tank; 19, small reboiler; 20, bottom product valve.

For the dynamic and control studies, computer control was used to control the temperature at the top of the packed distillation column. For this purpose, on-line computer control programmes were developed.

2.2. Method

In this work, optimal operating parameters for the control system of a packed distillation column were investigated and their effects on the overhead product composition of the packed distillation column were studied by using "Experimental Design" technique. Box–Wilson optimization technique [10] was used to determine the optimum overhead composition for the continuous distillation problem. Box–Wilson optimization method is based on experimental design. In the experimental design, the objective function is to make the overhead composition maximum and to determine the optimum values of reflux ratio and feed flow rate, when their values are between the highest and the lowest level. In this case only one set of optimal parameters was determined for maximum overhead composition, even if there are an infinite number of steady-states with the same distillate composition. To apply this model, the effect of the overhead product composition which is an output variable

Table 2 The matrix of the experimental plan

| Number of experiment, i | Code of parameters values | | | | Output variable |
|------------------------------|---------------------------|----------------------------|-------------------|---------------------------------------|--------------------|
| | $\varepsilon_{0,i}$ | ε ₁ | \mathcal{E}_2 j | ε_{1j} ε_{2j} | Υ; |
| | | | | | y_1 |
| | | | | | y_2 |
| 3 | | | | | y_3 |
| | | | | | y_4 |

was expressed as a first order regression model with interaction term.

$$
y = b_0 + b_1 x_1 + b_2 x_2 + b_1 2 x_1 x_2 \tag{1}
$$

where x_1 is the normalized parameter which is written as

$$
x_i = \frac{U_i - U_{iav}}{\Delta U_i}, \quad U_{iav} = \frac{U_i^+ + U_i^-}{2},
$$

$$
\Delta U_i = \frac{U_i^+ + U_i^-}{2}
$$

and $x_i x_{i+1}$ also a parameter and U_i the real parameter. U_{i} _{av} the average value of the real parameter. Where y the overhead product composition, *x*1, *x*² are operating parameters which are reflux ratio and feed flow rate values, respectively. b_0 , b_1 , b_2 , b_{12} are constants of the static linear model. According to experimental design method, to find the constants of the linear model and to apply optimization, 2^n experiment must be done. Where, 2 shows the highest and the lowest level of the operating parameters and the subscript "*n*" shows the number of operating parameters. In the present work, the matrix of the experimental plan is given in Table 2. Parameters of the first order linear regression model are determined as follows:

$$
b_i = \frac{\sum_{j=1}^{2^n} \varepsilon_{ij} y_j}{2^n} \tag{2}
$$

where ε_{ij} are codes of parameter values and they are given in Table 2.

In this work, heat-duty and feed temperature were chosen as the manipulated variable and disturbance, respectively, in the control work. That is why these two variables are not included in Eq. (1). The model which is given in Eq. (1), fits the column in the range of the highest and lowest values of feed flow rate, reflux ratio and overhead product composition. That chosen range can be changed when extreme differences occur in the optimal operation conditions. Optimal operating parameters of the process were found by using sharp slope optimization of Box–Wilson and a suitable regression model which is obtained from statical method [11].

3. Optimal adaptive generalized predictive control technique

Experimental design which is used in optimization method, includes some specific ranges of the variables which are changed between the highest and the lowest level. For every set of ranges of the variables, optimization provides a specific set point. When the set of ranges in the experimental design matrix is exceeded, optimization procedure chooses a new set of ranges which is included in the data bank, and optimization stage calculates the new optimal set point.

To establish the GPC algorithm it is supposed that a model of the linearized plant is expressed in terms of the following ARIMAX form

$$
Ay_t = Bu_{t-1} + \frac{C}{\Delta}e_t \tag{3}
$$

The function of the Δ operator ($\Delta = 1 - z^{-1}$) is to guarantee integral action in the controller which eliminate offset, i.e. a steady-state output disturbance. The polynomial $C(z^{-1})$ can then always be accepted as a stable polynomial since only the spectral properties of the signal $(C/\Delta)e(t)$ affect the predictions of future values of y_t .

The cost function to be minimized is

$$
J(u, t) = E\left\{\sum_{j=N_1}^{N_2} (y_{t+j} - r_{t+j})^2 + \lambda \sum_{j=1}^{N_u} (\Delta u_{t+j-1})^2\right\}
$$
(4)

where $\Delta u_{t+j} = 0$ for $j = N_u, \ldots, N_2$

 N_1 is termed the minimum costing horizon, N_2 the maximum costing horizon, and N_u the control costing horizon. The signal r_t is the reference signal which is chosen for the system output to track. The positive constant λ (control weighting) adds weight to the relative importance of the control and tracking errors. The expectation *E* is used in Eq. (4) to denote that the control values selected are estimated from data obtained up to and including time *t* and that a stochastic disturbance model has been assumed.

The quadratic minimization of Eq. (4), now corresponds to a direct problem of linear algebra with

$$
J = (\bar{y} - r)^T (\bar{y} - r) + \lambda (\bar{u})^T \bar{u}
$$
\n⁽⁵⁾

$$
\bar{y} = G\bar{u} + f \tag{6}
$$

where

$$
G = \frac{B}{A\Delta} \tag{7}
$$

$$
J = (G\bar{u} + f - r)(G\bar{u} + f - r) + \lambda(\bar{u})^T \bar{u} + \lambda(\bar{u})^T \bar{u}
$$
 (8)

where the future incremental control vector \bar{u} is given by minimizing *J* according to \bar{u} .

$$
\bar{u} = (G^T G + \lambda I)^{-1} (r - f) \tag{9}
$$

where $\bar{u} = \Delta u(t)$

In the present work, the steps used in the application of the GPC algorithm may be summarized as:

1. Apply PRBS signal and accumulate input and output data.

Fig. 2. Real-time optimization algorithm.

- 2. Estimate *A* and *B* from Eq. (3) using the Bierman U-D update algorithm.
- 3. Apply Eq. (9) to estimate the control signal and use this control signal at every sampling time.
- 4. Return to (1).

Real-time optimization algorithm is used to determine the optimum parameters of the packed distillation column. Block diagram of this algorithm is shown in Fig. 2. The overhead product composition, which is a function of adaptive regression model obtained from the algorithm was optimized on-line. The results obtained are compared with real data. When the variation of the values of the function is within acceptable limits, coefficients of adaptive model are evaluated by using parameter estimation algorithms. After that, optimization model is resolved. For unsteady-state condition, optimum values of the parameters are determined and given as a set point for the control system of the column. Thus, the function developed for the packed distillation column is not effected by the changes in operating conditions and this function is kept at its value by means of OA-GPC control. Estimation of the optimal parameters for real-time algorithms is based on adaptive static model [7].

4. Experimental and theoretical results

In the first part of this work, optimal continuous operating condition of a packed distillation column in a pilot plant was determined. Related operating parameters of the packed column were investigated and their effects on the overhead product composition which is related with temperature in the sense of equilibrium condition of the system were studied by applying Box–Wilson Method and Experimental Design technique. It was found that reflux ratio and feed flow rate have the most effect on the overhead product temperature. Box–Wilson Method was utilized to determine the optimal

Table 3 The matrix of experimental plan

| Number of experiment | R | $F \text{ (mol min}^{-1})$ | $T_{\rm D}$ (°C) |
|----------------------|---|----------------------------|------------------|
| | | 6.0 | 65.5 |
| 2 | | 2.2 | 65.2 |
| 3 | | 6.0 | 65.8 |
| 4 | | 2.2 | 65.0 |
| | | | |

values of these three variables which make the overhead product composition maximum. For the application of this method, the effect of overhead temperature of the packed distillation column as an output variable was expressed as a linear polynomial which is a first order regression model in Eq. (1).

$$
y = b_0 + b_1 x_1 + b_2 x_2 + b_1 2 x_1 x_2 \tag{1}
$$

where *y* is an overhead product temperature (or top product composition), x_1, x_2 are operating parameters which are reflux ratio and feed flow rate values, respectively. b_0 , b_1 , b_2 and b_{12} are the constants of the regression model. According to experimental design method, to find the constants of the related model and to calculate the optimal values of operating parameters, 2^n experiments must be carried out. Where 2 shows the highest and the lowest level of operating parameters and the subscript "*n*" shows the number of these parameters. In the present work, the matrix of experimental plan is given as in Table 3. The values of the constants for regression model were determined, as $b_0 = 65.395$, $b_1 =$ $-0.1025, b_2 = 0.235, b_{12} = -0.125.$

Optimal operating condition of the packed distillation column was calculated by using sharp slope optimization of Box–Wilson method. Table 4 shows the calculated optimum values of operating parameters for the packed distillation column. This optimal condition was also achieved experimentally as mentioned in the following sentences. In

Table 4 Optimum values of operating parameters

| $T_{\text{D, opt}}$ (°C) | $R_{\rm opt}$ | F_{opt} (mol min ⁻¹) | $T (^{\circ}C)$ |
|--------------------------|---------------|---|-----------------|
| 64.2 | 2.65 | 3.55 | 64.2 |

Table 5

Experimental optimal steady-state condition

Fig. 3. Open loop step response obtained using identified models, theoretical result and experimental data. Response of overhead temperature to a step decrease in the reboiler heat duty from 21200 to 19000 cal min−1.

the initial experimental work, the reboiler was filled with a methanol–water mixture at the feed composition. When the reboiler temperature reached the boiling temperature of feed composition, cooling water was sent to the condenser. The column was operated for approximately 1 h at the total reflux. After the system reached the steady-state condition, preheated mixture was fed with optimal values of flow rate to the reboiler. At the same time the reflux ratio was adjusted

Fig. 4. PID control of the top temperature of the packed distillation column when a step decrease in feed temperature from 60 to 43◦C was applied using the reboiler heat duty as the controlling variable $(1BYT = 0.144$ kcal min⁻¹).

to the optimal value. Within short time intervals, product samples were taken and their compositions were recorded by computer. Similarly the time variation of temperature was observed with computer control system. When the system reached a steady-state condition, optimal temperature profiles and composition were achieved constantly. So optimal values of the overhead temperature were observed. The experimental optimal steady-state condition is given in Table 5.

All the dynamic and control studies have been examined using this optimal steady-state condition. When the system was in steady-state condition, different values and types of step change were given and the time variation of temperature was observed. A lot of analyses on the process dynamics are given in the previously published paper [12].

Clearly a primary control objective is to maintain the product stream compositions as near to the optimal operating values as possible in the face of load disturbances affecting the column. Thus it was necessary to control the overhead temperature of the packed column. It was decided to employ the heat duty to the boiler as the manipulated variable which gives good sensitivity and a rapid response.

Fig. 5. PID control of the top temperature of the packed distillation column when a step increase in feed temperature from 60 to 70° C was applied using the reboiler heat duty as the controlling variable $(1BYT = 0.144$ kcal min⁻¹).

Table 6 Comparison of performance criteria of GPC of top temperature of packed distillation column to step decrease in feed temperature from 60 to 43◦C

| IAE | Figure number |
|-------|---------------|
| 0.192 | |
| 0.163 | |
| 0.158 | Fig. 6 |
| 0.171 | |
| 0.195 | |
| | |

A second order polynomial which is sufficient to represent the denominator plant dynamics was used as a system transfer function of the form.

$$
(y_t - y_{t-1}) = \frac{B(u_t - u_{t-1}) + Ce_t}{A}
$$
 (10)

where,

$$
A = 1 + a_1 z^{-1} + a_2 z^{-2}, \ B = B_0, \ C = 1
$$

where A and B are the polynomials and they are described by Eq. (10). By using PRBS signal and Bierman algorithm [13], system parameters were determined and $a_1 = 2.71$, $a_2 = -0.0141$ and $B_0 = 0.07295$ were found and then these

Fig. 6. GPC control of the top temperature of the packed distillation column when a step decrease in feed temperature from 60 to 43◦C was applied using the reboiler heat duty as the controlling variable $(1BYT = 0.144$ kcal min⁻¹).

parameters were employed to calculate the control parameters. The time variation of the top temperature obtained from experimental data, computer simulation program and identification model in response to a unit step decrease in heat duty given to the reboiler is shown in Fig. 3. Agreement between the identified model and simulation result is satisfactory enough. So it was decided to be used for controller design in the case studied.

The GPC and PID control of the overhead temperature were examined in terms of several load disturbance rejection. For PID control work control parameters were calculated by applying YS increase gain approach [4] after several tuning methods such as [3,14,15]. These parameters are $K_C = 19.19$, $\tau_I = 22.5$ min and $\tau_D = 1.17$ min. Results of two different step changes applied in PID control work are shown in Figs. 4 and 5. Experimental and theoretical control results using the same control parameters are shown in the same figures. Very good agreement was achieved between experimental and theoretical work.

The GPC algorithm is discussed in the previous section. When compared with PID control, the GPC provides a substantial array of tuning knobs. (a) The minimum costing horizon N_1 , (b) The maximum costing horizon N_2 , (c) The control costing horizon N_u , (d) The control weighting λ . All

Fig. 7. GPC control of the top temperature of the packed distillation column when a step increase in feed temperature from 60 to 70◦C was applied using the reboiler heat duty as the controlling variable $(1BYT = 0.144$ kcal min⁻¹).

these have suggested a 'default' set of tuning parameters which provide reasonable control in most applications, i.e. $N_2 = 10$ (or the plant rise time, which is ever the greater), $N_1 = 1$, $N_u = 1$, and $\lambda = 0$. In this work, the default values of N_1 , N_2 and N_u are used and λ varied in order to obtain the best result. Typical results for λ are shown in Table 6. After several tuning with λ value, suitable value of this parameter is taken as $\lambda = 1.2$. In all GPC control work the best value of λ and identified ARIMAX model were used to keep the overhead temperature at the optimal values. All the experimental and theoretical results are shown in Figs. 6 and 7 as well as the time variation of the manipulated variable. As it is seen that GPC control provides marginally better control than PID system. GPC control in Figs. 6 and 7 comes to set point more quickly than the PID control in Figs. 4 and 5. GPC control of the top temperature of the packed distillation column is marginally better than the PID control shown in both disturbances.

5. Conclusion

In the optimal operating conditions, the packed distillation column was controlled by using one of the self-tuning strategies which is called GPC. ARIMAX discrete time model was used to determine the system. In this model, the coefficients of *A* and *B* polynomials were determined with a parameter estimation technique which uses the reboiler heat duty as the manipulated variable which is input value and overhead temperature is used the controlled variable. The PRBS signal was given to the reboiler heat duty and then overhead temperatures were monitored. By using these input and output values, ARIMAX type model parameters of the system were evaluated from Bierman algorithm. These *A* and *B* system model polynomials with known coefficients were used in GPC algorithm. The system overhead temperature control was achieved with GPC algorithm in the face of various step changes which were given to feed temperature. The experimental and the theoretical results were compared with each other. It is shown that the results were in very good agreement. The GPC control is superior in all the cases studied. However, the PID control performance differs according to the type of disturbance given to the column. In some cases in which the disturbances are given to the feed temperature, the PID results are comparable to the GPC results, but in other cases when the disturbances are given to feed composition the PID performance is worse than the GPC control [8].

References

- [1] W.L. Bolles, J.R. Fair, Performance and designing of packed distillation columns, Inst. Chem. Eng. Symp. Ser. 56 (1980) 35–89.
- [2] M. Yuwana, D.E. Seborg, A new method for on-line controller tuning, AIChE J. 28 (1982) 433–440.
- [3] A. Jutan, E. Rodriguez, Extension of new method for on-line controller tuning, Can. J. Chem. Eng. 26 (1984) 802–807.
- [4] A.P. Wardle, S.M. Heatchock, The self-tuning control of multicomponent distillation columns, in: Proceedings of the International Conference on Intelligent Control and Instrumentation, Singapore, 1992, pp. 317–322.
- [5] K.J. Astrom, Theory and applications of adaptive control-A Servey, Automatica 19 (1983) 71–486.
- [6] V. Pamuk, M. Alpbaz, S.R. Asgharpour, The dynamics of packed distillation column distillating the mixture of water-methyl-alcohol under the effect of several disturbances, Doğa-Tr. J. Chemistry 14 (1990) 226–253.
- [7] D.W. Clarke, C.I. Mohtad, P.S. Tuffs, Generalized Predictive Control Part I. The Basic Algorithm, Automatica 23 (1987) 137–148.
- [8] S. Karacan, H. Hapoğlu, M. Alpbaz, Generalized predictive control to a packed distillation column for regulatory problems, Comput. Chem. Eng. 22 (1998) S629–S632.
- [9] S. Karacan, Application of optimal adaptive generalized predictive control to a packed distillation column, Ph.D. Thesis, Ankara University, 1997.
- [10] G.R. Box, K.B. Wilson, On the experimental attainment of optimum conditions, J. R. Statistical Soc. 13 (1951).
- [11] K.J. Astrom, Theory and applications of adaptive control-A Servey, Automatica 19 (1983) 71–486.
- [12] S. Karacan, Y. Cabbar, M. Alpbaz, H. Hapoğlu, The steady-state and dynamic analysis of packed distillation column based on partial differential approach, Chem. Eng. Processing 37 (1998) 379–388.
- [13] G.J. Bierman, Measurement Updating Using the U-D Factorization, Automatica 12 (1998) 375–382.
- [14] G.H. Cohen, G.A. Coon, Automatic Control of Processes, International Texbook Company, PA, 1967 pp. 345–356.
- [15] J.B. Ziegler, N.B. Nichols, Optimum settings for automatic controllers, Trans. ASME 64 (1942) 759.